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Muslim Logicians on Quantification of Predicate vs. Hamilton's View

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Abstract

According to Muslim logicians, the quantifier, in categorical logic, shows the quantity of the individuals of the subject in a statement; so its place is before the subject. Hence, if it comes before the predicate there arises some deviation in the main form of the statement, and such a statement is called a "deviant statement" (al-qaḍiyah al-munḥarifah). In modern logic, by contrast, the main characteristic of a predicate is being general or unsaturated and since a predicate has a propositional function, i.e. has free variables (or arguments), it can or should be quantified; hence, putting the quantifier before the predicate is consistent with the conditions and rules on constructing a well formed statement. Among contemporary logicians Hamilton is famous for his claim that predicates should also be quantified just like subjects. The viewpoints of Muslim and modern logicians, concerning the place of the quantifier in a statement, seems to be conflicted. Among Muslim logicians, Avicenna is the one who considers no problem in using such statements, although he calls them "deviant", and gives an explanation and analysis for them. In this paper, I have examined these views and shown that the conflict may be superfluous if Muslim logicians' approach to predicates is extensional, which, of course, can hardly be attributed to them.

Keywords: Deviant, Statement, Quantifier, Muslim logicians, Avicenna.

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Introduction

According to Muslim logicians, in the statement "A is B", provided that "A" and "B" are general terms, what is meant by "A" is those individuals which are A (or fall under "A") and what is meant by "B" is some concept or property, not individuals (it may be called "intensional approach" to predicate); because, if it were the individual which we mean by "B", then that individual is either the same as the individual meant by "A" or not; the former, yields a tautological statement and, hence, necessary true; and the latter, yields a self-contradictory statement and, hence, a necessarily false one, while our presupposition is that the statement "A is B" is contingent (Rāzī, p.130)

By this *reducto ad absordum* argument, the ancient logicians conclude that it is the concept or property, not the individual that is meant by predicate; on the other hand, the role of a quantifier, in first order (categorical) logic, is to show the quantity of the individuals; so the quantifiers cannot come before the predicate, otherwise some *deviation* will occur.

Almost all Muslim logicians who have dealt with the above topic have such a claim and in this paper I confine the discussion to three of them, by way of example. Shahrazūrī says "The quantifier is to show the quantity of the individuals of the subject" (Shahrazuri, 1383, p.114); and, by the above reason, he says nothing about the quantification of predicate. Ṭūsī, also, says "The quantifier determines the quantity of the judgment (judgment is about all or some of the subject)"(Ṭūsī, 1376, p.126). The parenthesis in Ṭūsī's speech, in fact, clarifies the vagueness in the expression "quantity of the judgment", and shows that what is meant is the amount of the individuals of the subject (Avicenna's view will be discussed in detail below).

In modern logic, however, if a term is to be quantified it should be "general", and its place in a sentence is indifferent, i.e. whether it is as a subject or a predicate. The reason is that a general term is unsaturated and predicative, so that if it is to be saturated there are two ways in doing so: satisfying it by a singular term; quantifying it by a quantifier.

We observe, therefore, that placing a quantifier before a predicate, in modern logic, not only does not make the statement deviant, but it is the very proper place of it. So, it seems, with respect to quantification of the predicate, there is an obvious conflict between the viewpoints of Muslim and modern logicians. In the following sections, further explanations regarding Muslim logicians' and especially Avicenna's views will be given by using, sometimes, symbolization of modern logic; and finally we can find how the ancient view differs from the modern view by the difference in their approaches to the logical role of the predicate

1. Further explanation of Muslim logicians' view

In some other detailed explanation we find the following notes from Avicenna: he says if the subject is a singular individual, then it is not quantified; to say "Every Zeid is so and so" is not true (or correct) (Avicenna, 1405, p.54); and here Avicenna uses the word "هذر" ("hadhir"); literally, it means the speech which is not considered (is unnoticed, unintended); but what more does Avicenna mean by this term? Is the statement meaningless or false or valueless? Avicenna continues and says that the singular predicate is, also, not quantified; for example, to say "this hand is all of this finger (or some of this body) is not true [correct], since what is here meant by the quantifier is not every individual, one by one, but the *whole* [in the case of universal quantifiers] or *part* [in the case of particular quantifiers]; however, what we usually mean by quantifier is not the whole, but *every one* "(ibid., pp.54-5). On the other hand, according to Avicenna, although saying "Zeid is all this person" is hadhir and unnoticed, but the contradiction of it, e.i. "Zeid is none of this person" is correct and true; however, this statement is true provided it is meaningful; and if a statement is meaningful, logically, its contradiction is also meaningful; therefore, we can conclude that what Avicenna intends by using the word "hadhir" is something which is "useless". So far, ancient and modern views are similar and the same: it is not true or correct to quantify a singular term, and what can be quantified is a general word.

By using the formalization in modern logic, we can better describe the ideas of Muslim logicians: if by putting the quantifier before the predicate we want to refer to those individuals who fall under the subject, then the quantifier which is before the subject is sufficient, i.e. no quantifier before the predicate is needed. In the language of modern logic the statement "Every A is B" will be symbolized as $(x)\ (Ax \supset Bx)$. And if those individuals under the predicate are different from those under the subject, so that we should have two different quantifiers, then we have $(x)Ax\supset(y)By$. However, in this case, we don't have a categorical statement, but a conditional one (since, each side of the conditional connective is itself a sentence not a propositional function), while we wanted from the beginning to analyze the categorical statement.

2. Why the name "deviant"

Shahrazūrī says "When the quantifier is before the predicate, it becomes a part of the predicate and the statement deviant from its natural form" (Shahrazūrī, 1383, p.155). Ṭūsī, also, says "The proper role of the quantifier is to determine the position of the judgment. If the quantifier resides before the predicate, which is the object of the statement, it conflicts with the main meaning of itself. The quantifier, therefore, in this case is only linguistically a quantifier [not really]" (Ṭūsī, 1376, p.126).

What Ṭūsī means by the position of judgment, it seems, is the same as the subject of the statement, since judgment is about the subject; hence, the quantifier should be before the subject so that the position of the judgment be correctly determined; so, if it comes before the object [i.e. predicate] it is not resident in its proper position and cannot do its role in the statement; and , it seems, this is what Ṭūsī means by saying, "in this case [it]is only linguistically a quantifier", that is , it is not a logical quantifier, but only a linguistic quantifier. On the other hand, it seems that, Avicenna prefers to call such statements "deviant" because of the change in position or role of the predicate: the previous predicate, in this case, becomes part of the new predicate and by itself it is no longer predicate and this is the root of deviation (Avicenna, 1405, pp.64-5).

Of course, we may say that both views are correct: quantifier and predicate both have changed their condition so that according to Muslim logicians some deviation has occurred.

3. Avicenna

The Muslim logicians, generally, don't see any (or much) use to put the quantifier before the predicate. However, Avicenna considers it useful. According to him, when the quantifier is before the predicate, the predicate is, in fact, constructed of both that quantifier and the object of the sentence, so that each of them is a *part* of the predicate (ibid., p.62). For example, in "All A is some B" the predicate *is not* "B", but "some B". One important thing, here, we should notice: even when the predicate is quantified, Avicenna emphasizes (ibid., pp.55 & 57 & 59) that the quantifiers should be interpreted not collectively (i.e. the quantifier does not make a collection or a class), but distributively (i.e. the individuals of the quantifier, one by one, are considered). In Shifā, Avicenna says that if bringing a quantifier over the object is useful, then you may use it as if the quantifier is not there; there is no difference whether the quantifier is prefixed to the object (i.e. predicate) or not; what is important is whether the statement *is true* or not (ibid., pp.62-3) (emphasis mine). He, then, continues to say that according to different modalities (i.e. necessity, possibility and impossibility), we would have different statements with different truth-values, and he begins to assess in details all AIEO Aristotelian statements with quantification of the predicate (Ibid. pp.59-60). Here, I have shown the detail of his speech for the universal affirmative statement (i.e., the A statement) in the following table (of course, Avicenna doesn't specify such a table, but we can construct it from his sayings)

Quantified Predicate / **Modal**	Universal affirmative	Universal negative	Particular affirmative	Particular negative
necessity	Every man *is every* animal (false)	Every man *is not any* animal or laughter [ḍāḥik] (false)	Every man *is some* animal or laughter (true)	Every man *is not every* animal or laughter(true)
impossibility	Every man *is every* stone (false)	Every man *is not any* stone (true)	Every man *is some* stone (false)	Every man *is not every* stone (true)
possibility	Every man *is every* writer [kātib](false)	Every man *is not any* writer (false)	Every man *is some* writer (false)	Every man *is not every* writer (true)

Explanation of the table: the rows in the above table show the four moods which a quantifier may have before the predicate, i.e. universal affirmative (A), universal negative (E), particular affirmative (I) and particular negative (O). And what is meant by "modal" in the first column is the modality of the object of the statement which now becomes *a part* of the predicate: for example, when we have “Every man is not any animal”, this statement is written in the row of “necessity” in the above table; but we should notice that the relation between “animal” and “man” is necessary while “animal”, as Avicenna stresses, is *a part* of the predicate; the predicate, in this example is "not any animal" which has the relation of impossibility to the subject (i.e. "man")(ibid., p.64); and we should be careful that the modality of a statement is that modality which occurs between the subject and the whole predicate (i.e. *de dicto* modality); therefore, the mentioned modality in the first column of the table shows the relation of the subject and a part of the predicate. The whole table is constructed for only universal affirmative statement concerning the subject, so that all the sentences begin with the word "every". In the case of particular negative instead of using the word "some" the expression "not every" is used so that the very sayings of Avicenna are translated.

Now we confront the question that, according to Avicenna, if putting the quantifier before the predicate does not yield any problem, what would be the relation of those individuals falling under the subject and those falling under the predicate? This relation, it seems, would have a better explanation by using symbolization of modern logic: as an example, the sentence "Every A is every B" would be shown as:

$(x)\ \{Ax \supset (y)\ [By \supset (x=y)]\}$

It says that, for every x, if x is A then for every y, if y is B, then x is identical with y; it is obvious that the statement is false.

In the case of particular affirmative predicate "Every A is some B", we have $(x)\{Ax \supset (\exists y)[By \wedge (x=y)]\}$; and in the case of universal negative predicate "Every A is not any B", we have $(x)\ \{Ax \supset (y)[By \supset (x \neq y)]\}$; and if in the case of particular affirmative the sign of equality changes to inequality we get symbolization of particular negative predicate.

An important question is whether this kind of analysis can be accepted by Avicenna and his followers? One corollary of such an analysis is that, we may have *nested quantifiers*; and if

Muslim logicians accepted such an analysis they would subconsciously think about nested quantifiers, so that such a relation will not be innovation of modern logic. Another corollary is that the relation of "inclusion", which is meant by copula “is” in AIEO statements, can be reduced to conditional and equality connectives, as is shown above.

I think the answer to the above question is negative: the approach to the above analysis invokes considering subject and predicate as some kind of classes and what would be important is the relation between the members (or individuals) of those classes (this can be called “extensional approach” to the predicate). But as was mentioned, Muslim logicians, including Avicenna, have an intensional approach to the predicate, and this is obvious from Avicenna’s emphasis that the quantifier attached to the predicate is part of it and is not independent and also the whole quantified predicate should be interpreted distributively. It seems that he wants us to think of, for example, “every B” or “some B” as a *unique* general concept, so that when we have “Every A is some B” it should mean that every one of the individuals falling under “A” has the property of being “some B”, and it does not mean that every one of those individuals is identical or equal to someone (in fact, to herself). Hence, for example, in the statement “Every man is some animal” the predicate is “some animal” and it is a general expression so that it can be attributed to some individuals, and the statement means every human being has the property of being some animal. In this interpretation the word “is” may still be a copula and denotes the inclusion of subject in predicate; for the case of our example we can imagine that the subject “man” is included in the predicate “some animal”, and we can imagine the predicate to have more extensions, e.g. horse and cow and lion and etc. On the other hand, we do not see the same declaration as the quantification of predicate about the quantification of subject: we don’t find any utterances from Avicenna or other Muslim logicians in which they have said the whole subject of a statement –contrary to the predicate – consists of the quantifier and the object of it, so that the quantifier and the quantified word together make the subject and each of them is one *part* of it. The reason is that their approach to the subject is extensional so that the target of quantifying the subject is alluding to those individuals falling under it. Another point deserving to be emphasized is, as was mentioned, the repetition of Avicenna concerning the interpretation of quantified predicate; generally, a quantifier can be interpreted collectively or distributively; in the case of the universal quantifier perhaps if we choose "all" for collection and "every" for distribution, it would make more sense; when we say "all A is all B" it means that the collection or set or class of A and B have the same members and these classes are equal or identical; and if we say "every A is all B" we attribute the whole class of B to every individual of A. In the case of particular quantifiers (i.e., "some") there can also be two interpretations (i.e., collection and distribution) but we do not find any explanation from Avicenna especially when the predicate term is general; the only explanation is where the term is singular: saying "some Zeid" means some organs of Zeid, for instance, his head or hand etc (Avicenna, 1405, p.54); in this use "some" means a *part* of something, not an individual; however, Avicenna has a uniform claim concerning both universal and particular quantifiers; in both cases the quantifiers are used distributively; the individuals are intended one by one, neither the set or class of them nor the their parts are intended (ibid.55). Now, for example, in saying "Every man is all animal" it is plainly false if we attribute "being all animals" to every individual man and if we say "Every man is some animal", it can be true if the unique concept of the predicate (i.e. "some animal") is attributed to every one of human beings (and this is the distributive interpretation of "some"); but if "some animal" has a collective sense (for example among the animals two of them, human and horse, are intended collectively) the result is a false statement, since every man is not at once both human and horse (unless, the class of animal has only one member, the human being, so that "some" refers only to it; in this case the statement is true even the interpretation of particular quantifier is collective). I think the main reason for selecting distributive interpretation of quantifiers, according to Muslim logicians, is that in saying for example "Every A is some B" (if it is true) what is actually claimed is that *the very* individuals which fall under A (aqd al-vaḍ*ʻa*) has the property of being some B (aqd al-ḥaml), so that in both situations (aqdain) the same *individuals* and each one of them are under discussion; now if the quantified predicate is interpreted in collective sense we have the *totality* of individuals, not individuality of them one by one under discussion and this is what is rejected by Muslim logicians.

Now if we are to show Muslim logicians' view by using modern logic symbolization, we must be careful to choose a single symbol for the quantified predicate. For example, the statement "All A is some B" may be symbolized as (x) (Ax ⊃ QBx) ("QBx" is assigned to the predicate "some B". "Q" may be the abbreviation of "Quantified Predicate" and its meaning differs in each case: it may mean "every B", "some B", "not any B", "not every B", respectively, due to the kind of quantification of the predicate, i.e. AIEO). In this type of symbolization, however, the quantifiers prefixed to the predicate are not reflected. Of course we can choose another symbolization to remedy this defeat; for example each letter of the standard symbol AIEO may be prefixed to the predicate by convention:

"Every A is Every B" is shown as $(x)\{Ax \supset ABx\}$

"Every A is some B" is shown as $(x)\{Ax \supset IBx\}$

"Every A is not any B" is shown as $(x)\{Ax \supset EBx\}$

"Every A is not every B" is shown as $(x)\{Ax \supset OBx\}$

In the case of "Every A is every B", it seems that the predicate "every B", although being a universal expression, has no extension and can be accounted as an empty expression. For example, what is or can be that individual which is "every animal" i.e. has the property of being every animal? And because of this, it seems, all of the instances of "Every A is every B" are false (according to some modern logicians, especially Hamilton, this statement can have true instances, of course if we consider special interpretation for quantifier and copula; we have some reference to this issue in the following section).

4. Some historical notes

Since the issue of "deviant statement" is directly related to the quantification of the predicate, historically, we should look for such quantification among ancient logicians. Avicenna says that Aristotle does not deal with deviant statements (Ibid., p.65), perhaps because syllogisms usually do not have such statements as their premises..The most famous logician of the medieval ages who has considered such statements is John Buridan (14th century) and he has introduced the "distributed term" by which some rules are given for testing the validity of syllogisms (Gensler,2010, p.356). A distributed term is a universal term whose concept is attributed to all individuals falling under it. It is obvious that among AIEO statements the subject terms of universal (affirmative and negative) statements are distributed and the subject terms of particular statements are undistributed. But what is unclear is the situation of predicates in these four statements. Buridan explains that negative statements have distributed predicates, and affirmative ones have undistributed predicates. On the other hand, a distributed term is such a term that a universal quantifier can be put before it and, by contrast, a particular quantifier can be put before undistributed term. So the AIEO statements can be rephrased and we can, respectively, have: "All A is Some B", "Some A is some B", "No A is any B" and "Some A is not any B".

Today Buridan's idea is widely accepted and his rules of validation of Aristotle's syllogism are taught to logic students and most of the classical (traditional) books have devoted certain sections to them, and these statements are no longer considered as deviant.

5. Hamilton's view

The most famous mathematician and logician in nineteenth century who has dealt with such issue is Sir William Hamilton Bart. Of course, he was under the influence of mathematicians before him such as Leonhard Euler and J.D.Gergonne (Kneale, pp.349-50). In 1761 Euler tried to draw different relations between subject and predicate in a statement by using geometrical pictures; he came to the result that by using circles, three pictures can show five relations: when the circle A is inside the circle B we have the statement "All A is B"; when the circles are out of each other we have "No A is B"; and the intersection of the circles can show three statements: "Some A is B", "Some A isn't B" and "Some B isn't A"; of course, in Euler's view "some" means "some but only some". It was obvious that the approach of Euler to the analysis of the subject and predicate was extensional, and he considered them as classes. Fifty years after Euler, Gergonne, a French mathematician, tried to develop Euler's idea and this time he used five different symbols to show those five relations (ibid. p.350); he was convinced that these five relations are exhaustive and exclusive. Their approach was followed and developed by Hamilton; In 1846, in his work – New Analytic of Logical Forms – Hamilton tried to recognize eight forms of statements not only for Aristotelian syllogism but also for other sciences (ibid. p.253); in this approach the relations of individuals in different classes are under study and in this case quantifying subjects or predicates makes no difference, since both of them are considered as classes. This view was mainstream for the innovation of set theory. According to Hamilton, there are eight (instead of four) quantified statements, since each one of the AIEO statements can have universal and particular predicates. Hence, the eight quantified statements are (Heath, 1976, pp.447-8):

1-All A is all B

2-All A is some B (the famous "A" statement)

3-some A is all B

4-some A is some B (the famous "I" statement)

5-Any A is not any B (the famous "E" statement)

6-Any A is not some B

7-Some A is not any B (the famous "O" statement)

8-Some A is not some B

Hamilton's analysis concerning the terms in a proposition is somehow opposite to the orthodox Aristotelian view. He says:

"1- The terms of a proposition are only terms as they are terms of relation; and the relation here is the relation of comparison.

2- As the propositional terms are terms of comparison, so they are only compared as quantities- quantities relative to each other. An affirmative proposition is simply the declaration of an *equation* [my italic], a negative proposition is simply the declaration of a non-equation, of its terms…

3- The quantity of the proposition in conversion remains always the same; that is the absolute quantity of the converse must be exactly equal to that of the convertend. It was only from overlooking the quantity of the predicate… that two propositions exactly equal in quantity, in fact the same proposition, perhaps, transposed, were called the one *universal*, the other *particular*, by exclusive reference to the quantity of the subject." (Hamilton, p.259-60)

As can be seen, Hamilton's approach to the interpretation of ingredients of a proposition is extensional i.e. what we mean by both subject and predicate is their individuals. In item 1, he considers the *only relation* between subject and predicate to be the relation of comparison, by which he means their individuals are compared to each other; and the result of this comparison is asserted in item 2: either the collective of the amount of the individuals are the same or not, that is either these classes are equal or not; so what is meant by "is" in a proposition is the relation of *equality*, against Aristotelian view in which there are two more relations (based on different cases): inclusion and membership; one of the corollary of considering "is" as equality, which is mentioned in item 3, is that calling one term as "subject" and the other as "predicate" is somehow superfluous, because the two sides of equality can be changed and replaced by each other without changing in the content of the proposition; and since in this replacement the quantifier of each term should also transfer together with that term the total (absolute) amount of quantifiers remains the same; for example if we have "All A is some B", this statement is tantamount to "all A = some B"; now in replacing the two sides by each other the quantifiers, too, will be transfered so that we have "some B=all A", and the content remains the same; now which term is subject and which is predicate? It seems that Hamilton answers that no difference and each terms may be considered as subject or predicate (ibid. p.278); also in both statements (actually we have only one statement, because of the sameness on the content) the absolute amount of quantifiers is the same: each one has a universal and particular quantifiers; so, in Hamiltons's view, whether a universal quantifier or a particular quantifier is at the beginning of the statement so that that statement is considered as a universal or particular respectively, is also superfluous and it makes no difference, again opposite to Aristotelian view. Another corollary is that the relation of conversion will be superfluous and we don't have logical difference between the main statement (which is called "convertend" by Hamilton) and the converse statement; hence, in his view "Some B is all A" is the converse of "All A is some B" and vice versa. Many other corollaries may be expected concerning the square of opposition and other relations which I dismiss here.

Now about the above list some more things can be mentioned (Heath, 1976, pp.447-8):

If all of these eight statements are to be true, some curious results are obtained: in the case of the first statement "All A is all B", either the quantifier "all" is part of the predicate "B" or not; if it is not, then, it seems, that the statement contains within itself, not one statement, but two, namely, "All A is B" and "All B is A"; on the other hand, if it is part of the predicate (which Hamilton believes), then, here, we have a *singular* statement not a universal one; when we say "All men are all rational animals" the predicate no longer applies to the subject distributively, but collectively; since it is obvious that all of rational animals (i.e. human beings) cannot be attributed to each person. The meaning of the statement is, in fact, that the class of men and the class of rational animal are co-extensive. This statement also shows that "is", in this case, is not copula, but identity (or equality) and we saw that Hamilton himself emphasized on this matter. Another problem concerning the first statement is that it doesn't have any contradiction in that list; it seems that the proper candidate for being its contradiction is the last statement, but both "All A is all B" and "Some A is not some B" can be true; for example "All men are all rational animals" is true and "Some men is not some rational animal" is, also, true, because for example, this person is not that person. Now if we propose this constraint that two contradictory statements should have the same quantifier concerning the predicate so that the above statement would have the seventh statement of the above list as its contradiction, i.e. "Some A is not any B", then the statement "O" will no longer be contradictory to the statement "A", since they have different quantified predicate, a claim which nobody has accepted.

Another problem concerning the whole list of eight statements is that the various kinds of the relations between two classes can be shown by the first five statements of the list (1 to 5, in fact five Euler's relations) and, it seems, that the last three statements are redundant, namely their content are somehow included in others. Bednarowski tries to find some interpretation for these last three statements; he sometimes considers these statements as an alternative for some of the first five statements, sometimes as the complements of them and also some other interpretations; but generally he is not very satisfied with these interpretations (Bednarowski, 1956, pp.222-4).

6. Conclusion

According to Muslim logicians, the position of quantifier in a categorical statement is before the subject; and the predicate is not quantified and if it is quantified the previous predicate is no longer a predicate but is part of it, and also the quantifier is another part of the new predicate and hence, the situation and role of the predicate and quantifier have changed, and according to Muslim logicians, some deviation has occurred so that such a statement with quantified predicate is called "deviant statement".

In spite of most Muslim logicians, Avicenna considers no problem in using deviant statements provided that that statement is true, and he gives many examples with different modalities some of which are correct and true and some are false. If, on the other hand, the predicate is singular and quantified, e.g. "Zeid is all this person", according to Avicenna, the singular sentence would be false and "hadhir" (i.e., unnoticed, unintended). Its contradiction, namely "Zeid is none of this person" would be true. This statement, however, is true provided it is meaningful. Here this question arises that how it is *correct* to put the quantifier before the singular predicate in negative sentence, but doing the same thing in affirmative one is incorrect? One reply can be that the meaning of "hadhir" in Avicenna's word is not "meaningless" of the statement, but "being *useless*"; so, both of the above statements are meaningful, but the first one, contrary to the second, is useless, and hence "hadhir".

We may have different approaches to the content of the quantified predicate: intensional and extensional. By the intensional approach we mean that what is intended is the whole meaning of the quantified predicate, not the individuals under it, although its meaning is some property of those individuals under the subject of the statement. It seems that this approach is accepted by Muslim logicians. By the extensional approach we mean that what is intended is the individuals under the quantified predicate; of course, in this case the quantifier may be interpreted as showing the individuals collectively (which is Hamilton's view) or distributively.

Avicenna's and logicians' words, concerning the quantified predicate statements, can be analyzed better if we use symbolizations of modern logic. If their approach to the predicate were extensional, and also the quantifier attached to the predicate *is not part* of it (today, this is the orthodox view of first order predicate logic), then we could say that they had knowledge of nested quantification; and, also, the relation of "inclusion" in their analysis might be explained in terms of conditional and identity relations.

However, we cannot attribute such an approach to Muslim logicians, because they obviously stipulate that in a quantified predicate the quantifier is *part* of the predicate; this means that a quantified predicate, for them, is a kind of general expression and like other such predicate terms in an analysis of subject and predicate what is meant by predicate is its *concept* (not its individuals) i.e., some property of the individuals falling under the subject. So their approach is intensional and their analysis of quantified predicate is not the same as modern logic concerning the nested quantification.

However, many modern logicians, especially William Hamilton, like Avicenna, consider the quantifier as part of the predicate; but the difference between them and Avicenna is that their approach to the predicate is extensional not intensional; Hamilton considers both subject and predicate as classes and as the collection of individuals, just opposite to Avicenna who interprets quantifiers distributively. Hamilton's view has many corollaries which most of them are against the view of Muslim logicians: the relation in a statement between two terms is "equality"; calling one term "subject" and the other term "predicate" and also calling one statement "convertend" and the other "convers", all of these, are superfluous, etc. Hamilton's view is confronted with sever criticisms, especially it seems that all statements reduce to singular statement; he also extends four famous Aristotelian statements (i.e. AIEO) to eight statements by adding universal and particular quantifiers to predicate; it is not obvious whether the last three statements of his list have new content or not; also, the relation of contradiction between some of these statements is confronted with some challenge; for example, it is not obvious which statement should be the contradiction of the statement "All A is All B"; and also the combination of quantifier and negation has ambiguous interpretation.

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